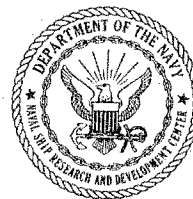


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# NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

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## DETERMINING THE DYNAMIC BULK MODULUS AND THE ELASTIC-LOSS FACTOR OF POLYMERS FROM ACOUSTIC MEASUREMENTS

by

Jan M. Niemiec

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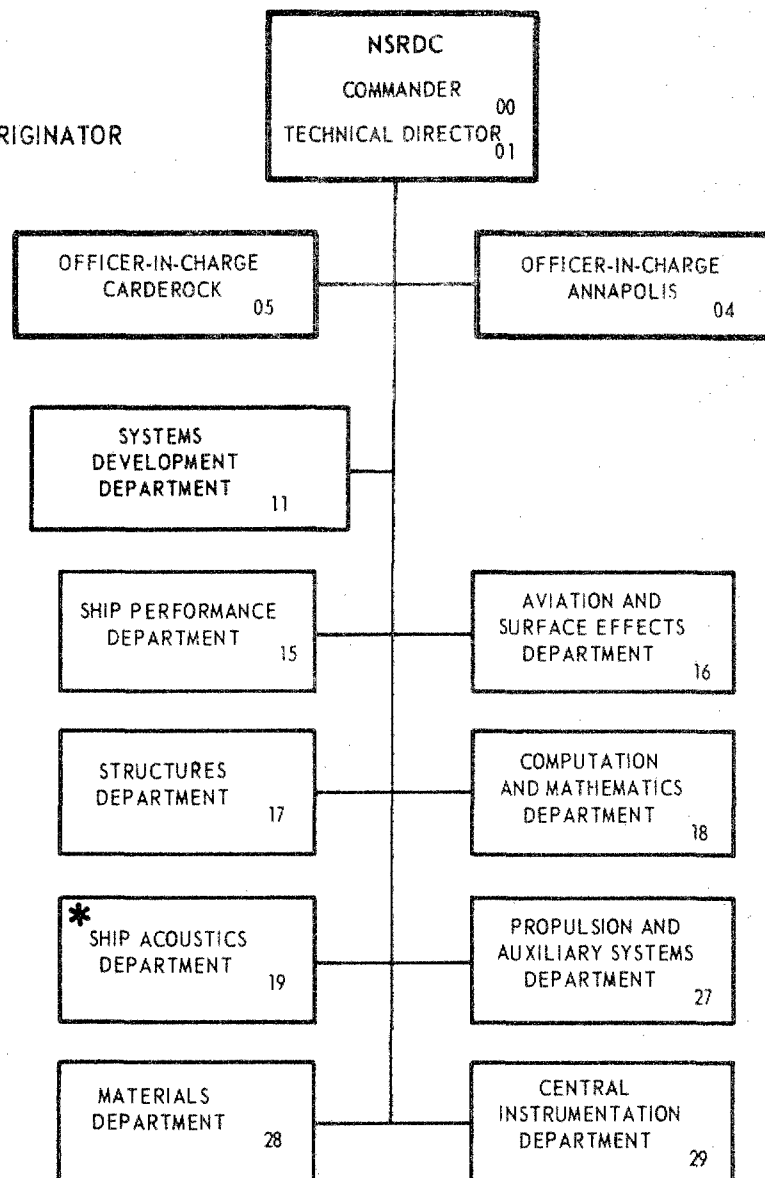
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ACOUSTIC MEASUREMENTS

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## ABSTRACT

A dynamic measurement technique is presented for determining the bulk modulus and the associated-loss factor of elastomeric and plastic materials. A resonant water column is used from which stiffness and damping are determined by the resonance frequency and the bandwidth of its resonance curve. A sample is inserted into the water column at a pressure maximum, and the resonance frequency and the bandwidth of the system are measured. The difference in the resonance frequency and the bandwidth of the water column without and with the sample present is then used to calculate the real component of the bulk modulus and the loss factor of the sample. Experimental results for various materials are presented which verify the theory.

## ADMINISTRATIVE INFORMATION

This work was supported by the Naval Ship Systems Command, Code 037, under Task Area SF 35452003, Task 01361, Project Element 62512N.

## INTRODUCTION

The basic theory for determining the dynamic bulk modulus and the associated loss factor from acoustic measurements of a resonant fluid column was first developed by Meyer and Tamm.<sup>1</sup> It was later used by Sandler<sup>2</sup> and Cramer and Silver<sup>3</sup> for bulk modulus measurements of elastomeric and plastic materials. Other investigators<sup>4,5</sup> have used a small pressurized resonant chamber to measure the dynamic compressibility of plastic materials and, therefore, the dynamic bulk modulus of these materials. The theory set forth here is basically similar to that developed by Meyer and Tamm; however, the measurement technique has been improved and simplified.

---

<sup>1</sup>Meyer, E. and K. Tamm, "Ein Akustisches Messverfahren zur Bestimmung der Dynamischen Stoffe," *Akustische Zeitschrift*, Vol. 2 (7 Jan 1942).

<sup>2</sup>Sandler, C.S., "An Acoustic Technique for Measuring the Effective Dynamic Bulk Modulus of Elasticity and Associated Loss Factor of Rubber and Plastics," U.S. Naval Ordnance Laboratory Report 1534 (1 Sep 1950).

<sup>3</sup>Cramer, W.S. and I. Silver, "Acoustical Properties of Rubber as a Function of Chemical Composition," U.S. Naval Ordnance Laboratory Report 1778 (Feb 1951).

<sup>4</sup>McKinney, J.E. et al., "Apparatus for the Direct Determination of the Dynamic Bulk Modulus," *Journal of Applied Physics*, Vol. 27, No. 5, pp. 425-430 (May 1956).

<sup>5</sup>Heydemann, P., "The Dynamic Compressibility of High Polymers in the Frequency Range from 0.1 Hz to 60 kHz," *Acustica*, Vol. 9 (1959).

## THEORY

The dynamic bulk modulus of a material can be expressed as a complex number of the form

$$B^* = B' + i B'' \quad (1)$$

where  $B^*$  is the complex modulus,  $B'$  is the real component, and  $B''$  is the imaginary component or "loss" modulus. If the material is sufficiently rigid,  $B^* = B'$ .

The loss modulus represents dissipation of energy into heat upon deformation of the material. The ratio of the energy dissipated to the total energy stored in the material defines the loss tangent as given by

$$\frac{B''}{B'} = \tan \delta \quad (2)$$

To simplify matters this loss tangent is often referred to as a loss factor

$$\tan \delta = \eta \quad (3)$$

By combining Equations (1), (2), and (3), the complex bulk modulus can be expressed as

$$B^* = B'(1 + i \eta) \quad (4)$$

Thus, if  $B'$  and  $\eta$  are known, the complex bulk modulus of the material is completely specified.

The relationship between the previously mentioned quantities  $B'$  and  $\eta$  and the resonance frequency shift and increased damping due to the insertion of a material sample into a resonant fluid column can be derived by considering the propagation of a plane acoustic wave in a rigid cylinder. In order to calculate the shift in resonance frequency and the increase in damping, let us assume that the sample is a thin layer, small, compared to the wavelength in the column, and exactly fitting the cross-sectional area of the cylinder as shown in Figure 1.

The characteristic acoustic impedance  $Z$  of the fluid is defined as  $Z = \rho_0 c$ . The solution to the plane wave equation subject to the boundary conditions  $p_1 = p_{10}$  and  $u_1 = u_{10}$  at  $x_1 = 0$  yields the acoustic pressure and the particle velocity in the lower half of the fluid column

$$p_1 \approx p_{10} \cosh \gamma x_1 - Z u_{10} \sinh \gamma x_1 \quad (5)$$

$$\text{if } |a| \ll |i \beta|$$

and

$$u_1 \approx u_{10} \cosh \gamma x_1 - \frac{p_{10}}{Z} \sinh \gamma x_1 \quad (6)$$

$$\text{if } |a| \ll |i \beta|$$

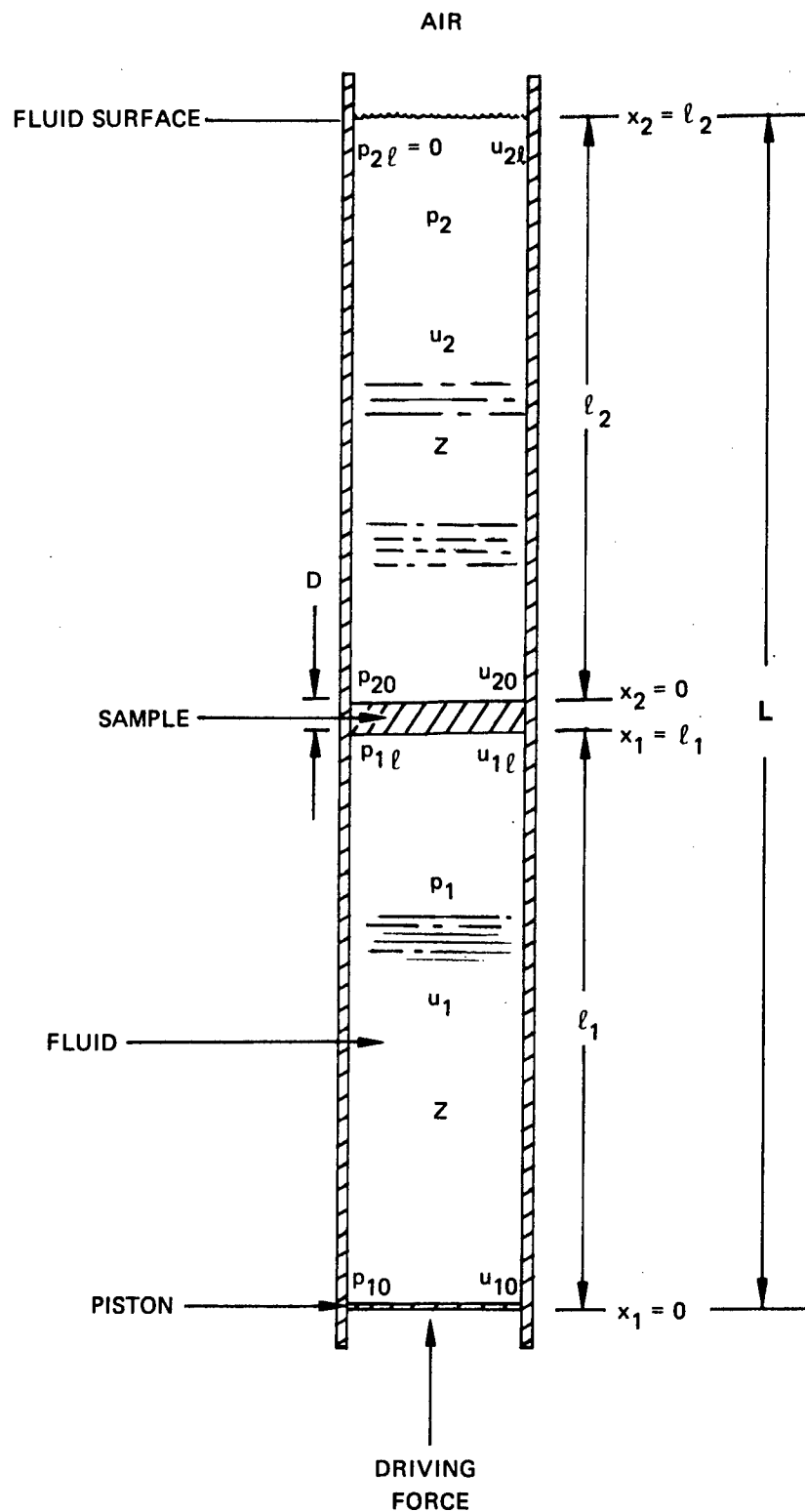


Figure 1 — Acoustic Transmission Line

The propagation constant  $\gamma$  equals  $\alpha + i\beta$ , where  $\alpha$  is the attenuation constant and  $\beta = \omega/c$  is the phase constant. The solution to the wave equation subject to the boundary conditions  $p_2 = 0$  and  $u_2 = u_{20}$  at  $x_2 = \ell_2$ , the air-fluid interface, yields the acoustic pressure and the particle velocity in the upper half of the fluid column

$$p_2 \approx Z u_2 \sinh \gamma (\ell_2 - x_2) \quad (7)$$

$$\text{if } |\alpha| \ll |\beta|$$

and

$$u_2 = u_2 \cosh \gamma (\ell_2 - x_2) \quad (8)$$

where  $u_2$  is the particle velocity at  $x_2 = \ell_2$ .

Assuming that the thickness of the sample is small, compared to the length of the column, and the acoustic pressures on the upper and lower faces of the sample are approximately equal, one finds (Appendix A)

$$p_1 = p_{20} = \frac{B^*}{i\omega D} (u_{1\ell} - u_{20}) \quad (9)$$

Substituting the expressions for  $p_1$  and  $p_{20}$  into the first part of this boundary condition,  $p_{1\ell} = p_{20}$ , we obtain

$$Z u_{10} = p_{10} \frac{\cosh \gamma \ell_1}{\sinh \gamma \ell_1} - Z u_2 \frac{\sinh \gamma \ell_2}{\sinh \gamma \ell_1} \quad (10)$$

Substituting the expressions for  $u_1$ ,  $p_{20}$ , and  $u_{20}$  into the second part of this boundary condition yields

$$Z u_2 \sinh \gamma \ell_2 = \frac{B^*}{i\omega D Z} (Z u_{10} \cosh \gamma \ell_1 - Z u_2 \cosh \gamma \ell_2 - p_{10} \sinh \gamma \ell_1) \quad (11)$$

Combining Equation (10) and Equation (11) and simplifying, we have

$$Z u_2 = \frac{p_{10}}{\sinh \gamma L + \frac{i\omega D Z}{2 B^*} (\cosh \gamma L - \cosh 2 \gamma A)} \quad (12)$$

where  $L = \ell_1 + \ell_2$  and  $2A = \ell_1 - \ell_2$ .



## WATER COLUMN WITHOUT SAMPLE

The particle velocity at the surface of the fluid column is given by Equation (12) as

$$u_{2\ell} = \frac{P_{10}}{Z \sinh \gamma L} \quad (13)$$

Assuming small damping, velocity resonance of the fluid column occurs at the frequencies  $\omega_0$  for which the imaginary part of  $\sinh \gamma L$  approaches zero

$$I_m (\sinh \gamma L) \approx \sin \frac{\omega_0 L}{c} = 0$$

or

$$\omega_0 = \frac{n \pi c}{L} ; \quad n = 1, 2, 3, \dots, \infty \quad (14)$$

In the neighborhood of resonance,  $\omega = \omega_0 \pm d\omega$ , where  $d\omega$  is a small change in frequency about  $\omega_0$ ; for small damping, we can assume that  $\sinh \gamma L = \pm a L \pm \frac{i d \omega L}{c}$ ; hence, the magnitude of the volume velocity is given by

$$|u_{2\ell}| = \frac{P_{10}}{Z \sqrt{(aL)^2 + \left(\frac{d\omega L}{c}\right)^2}} \quad (15)$$

At resonance ( $d\omega = 0$ )

$$|u_{2\ell}|_{\text{RES}} = \frac{P_{10}}{Z a L} \quad (16)$$

Since the acoustic power in the fluid column is proportional to the square of the particle velocity, we have

$$\frac{|u_{2\ell}|^2_{\text{half-power}}}{|u_{2\ell}|^2_{\text{RES}}} = \frac{1}{2} \quad (17)$$

Substituting Equations (15) and (16) into Equation (17), and defining  $d\omega = \frac{\Delta \omega}{2}$  at the half-power yields

$$\Delta \omega = 2 c a \quad (18)$$

## WATER COLUMN WITH SAMPLE

Insertion of the sample shifts the resonance frequency, so that the new resonance frequency is given by  $\omega_0' = \omega_0 - \delta\omega$ , where  $\delta\omega$  is the shift in resonance frequency. Substituting  $\omega_0'$  for  $\omega$  in Equation (12), and making the following assumptions in the neighborhood of resonance and for small damping,  $\sinh \gamma' L \approx \pm a' L \pm i \frac{\delta \omega L}{c'}$ ,  $\cosh 2 \gamma' A \approx \cos \frac{2 \omega_0' A}{c'}$ , and  $\cosh \gamma' L = \pm 1$ , Equation (12) then reduces to

$$Z_{u2} = \frac{P_{10}}{\pm a' L \pm i \frac{\delta \omega L}{c'} + \frac{i \omega_0' D}{2} \cdot \frac{Z}{2} \left( \pm 1 - \cos \frac{2 \omega_0' A}{c'} \right)} \quad (19)$$

where the positive signs apply for an even number of pressure antinodes, and the negative signs apply for an odd number. The primed variables refer to the appropriate quantities with the sample installed in the fluid column. For an odd number,  $A = \epsilon$ , the distance of the sample from a pressure antinode. For an even number of antinodes,  $A = \epsilon + \lambda/4$ . If we only consider an even number of antinodes

$$\frac{1}{2} \left( 1 - \cos \frac{2 \omega_0' A}{c'} \right) = \cos^2 \frac{\omega_0' \epsilon}{c'}$$

and Equation (19) then becomes

$$Z_{u2} = \frac{P_{10}}{a' b + i \left( \frac{\delta \omega L}{c'} + \frac{\omega_0' D Z}{B^*} \cdot \cos^2 \frac{\omega_0' \epsilon}{c'} \right)} \quad (20)$$

Substituting for  $Z$  and  $B^*$  in Equation (20), and letting  $c' \approx c$ —which is valid for resonance frequency shifts of less than 150 Hz—yields

$$Z_{u2} = \frac{P_{10}}{a' L + K \eta \cos^2 \frac{\omega_0' \epsilon}{c} + i \left( \frac{\delta \omega L}{c} + K \cos^2 \frac{\omega_0' \epsilon}{c} \right)} \quad (21)$$

where

$$K = \frac{\omega_0' D \rho_0 c^2}{c B' (1 + \eta^2)}$$

With the sample installed, velocity resonance will occur for frequencies at which the imaginary term in the denominator of Equation (21) vanishes. Furthermore, if the sample is positioned at a pressure antinode, then  $\cos^2(\omega_0 \epsilon/c) = 1$ , and the shift in resonance frequency is simply

$$-\delta \omega = K \frac{c}{L} \quad (22)$$

where the negative sign implies only a reduction in resonance frequency. Substituting for  $K$  into Equation (22), and rearranging terms, we obtain an expression for  $B'$

$$B' = \rho_0 c^2 \cdot \frac{f_0}{\delta f} \cdot \frac{D}{L} \cdot \frac{1}{1 + \eta^2} \quad (23)$$

An expression for the half bandwidth with the sample installed in the fluid column can be derived analogous to Equation (18); the result is

$$\Delta \omega' = 2 (ca + \eta \delta \omega) \quad (24)$$

However,  $\Delta \omega' = 2ca$  from Equation (18); therefore

$$\Delta \omega' = \Delta \omega + 2 \eta \delta \omega$$

or

$$\eta = \frac{\Delta f' - \Delta f}{2\delta f} \quad (25)$$

For some very lossy materials, the shift in resonance frequency may be excessive, greater than 150 Hz, when the sample is placed at the pressure antinode. In this case the sample can be placed away from the pressure antinode. If this is done, the factor  $\cos^2(\omega_0 \epsilon/c)$  remains in the expression for  $B'$ , so that

$$B' = \rho_0 c^2 \cdot \frac{f_0}{\delta f} \cdot \frac{D}{L} \cdot \frac{1}{1 + \eta^2} \cdot \frac{1}{\cos^2 \frac{\omega_0 \epsilon}{c}} \quad (26)$$

In general, one has to consider a sample of arbitrary shape that will not just fit the cross-sectional area of the cylinder. Ver Nooy and Cramer<sup>6</sup> have shown experimentally that the influence of

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<sup>6</sup>Ver Nooy, D.P. and W.S. Cramer, "Analytical Study of Resonant Tube Method of Measuring Dynamic Bulk Modulus," U.S. Naval Ordnance Laboratory TN 1124 (21 Jan 1952).

the sample on the resonance properties of the fluid column is independent of its shape and is equivalent to that of a uniform cross-sectional layer of the material. Thus, the thickness of the sample  $D$  and the length of the column  $L$  may be replaced by the volume of the sample  $V'$  and the volume of the column  $V$ , respectively, in the expression for  $B'$

$$B' = \rho_0 c^2 \cdot \frac{f_0}{\delta f} \cdot \frac{V'}{V} \cdot \frac{1}{1 + \eta^2} \quad (27)$$

It has been assumed until now that the walls of the tube are perfectly rigid. In actual practice the walls have compliance which must be taken into account.<sup>7</sup> This compliance results in a lower sound speed and, therefore, a lower resonance frequency of the fluid column. The reduced sound speed is easily obtained from the resonance frequency of the fluid column. This value for  $c$  is then used to calculate the stiffness of the fluid column.

## APPARATUS

A water-filled steel tube, having a wall thickness of 0.320 in., an inside diameter of 4.50 in., and a length of approximately 3 ft was used. The tube was supported vertically by a steel clamp at the velocity node, located one-quarter of a wavelength from the water surface, i.e., one-quarter of the length of the water column, and the bottom of the tube was sealed by a thin stainless steel membrane. The membrane was excited by an electromagnetic shaker, coupled to a force gage from which the output signal was held constant by means of a feedback circuit. The standing wave produced by the transducer was received by an active element hydrophone 0.1 in. in diameter from which the output signal was amplified, passed through a narrow band-pass filter, and indicated by a voltmeter (all conveniently done by a General Radio Type 1900-A Wave Analyzer). The water column was tuned to the resonance frequency by maximizing the output from the hydrophone. A schematic of the resonance tube is shown in Figure 2 and a block diagram of the measurement circuit is shown in Figure 3. The water column was tuned to the second harmonic frequency of approximately 1550 Hz. The half-power bandwidth of the resonance curve was 1.2 Hz; hence, the  $Q$  of the water column was approximately 1300 which indicates very small damping. The resonance peak was sharp and well defined for the materials tested. This made possible very precise measurements of resonance frequencies.

A preliminary study of the sound-pressure distribution of the standing wave in the water column showed that the pressure changed less than 3 percent within  $\pm 0.50$  in. of the pressure antinode; hence, samples of as much as 1.0 in. in thickness were tested. Although the shape of the sample does not affect the measurement,<sup>6</sup> we found it convenient to test disk-shaped samples of various thicknesses.

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<sup>7</sup>Richardson, E.G., "Technical Aspects of Sound," Vol. II, Elsevier Publishing Company, New York, p. 208 (1957).

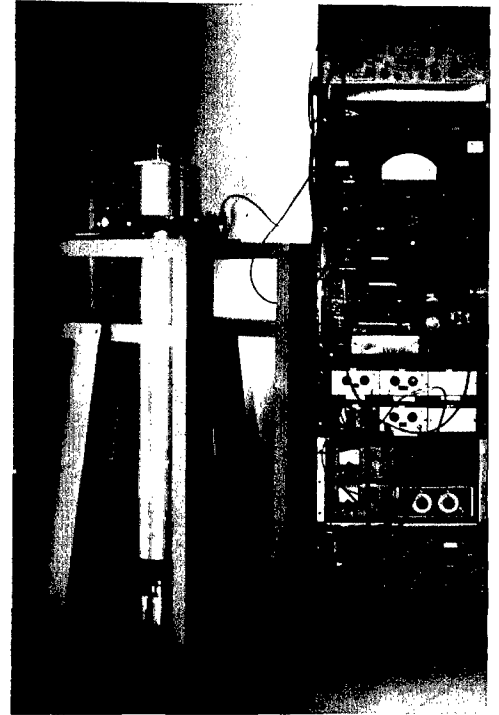
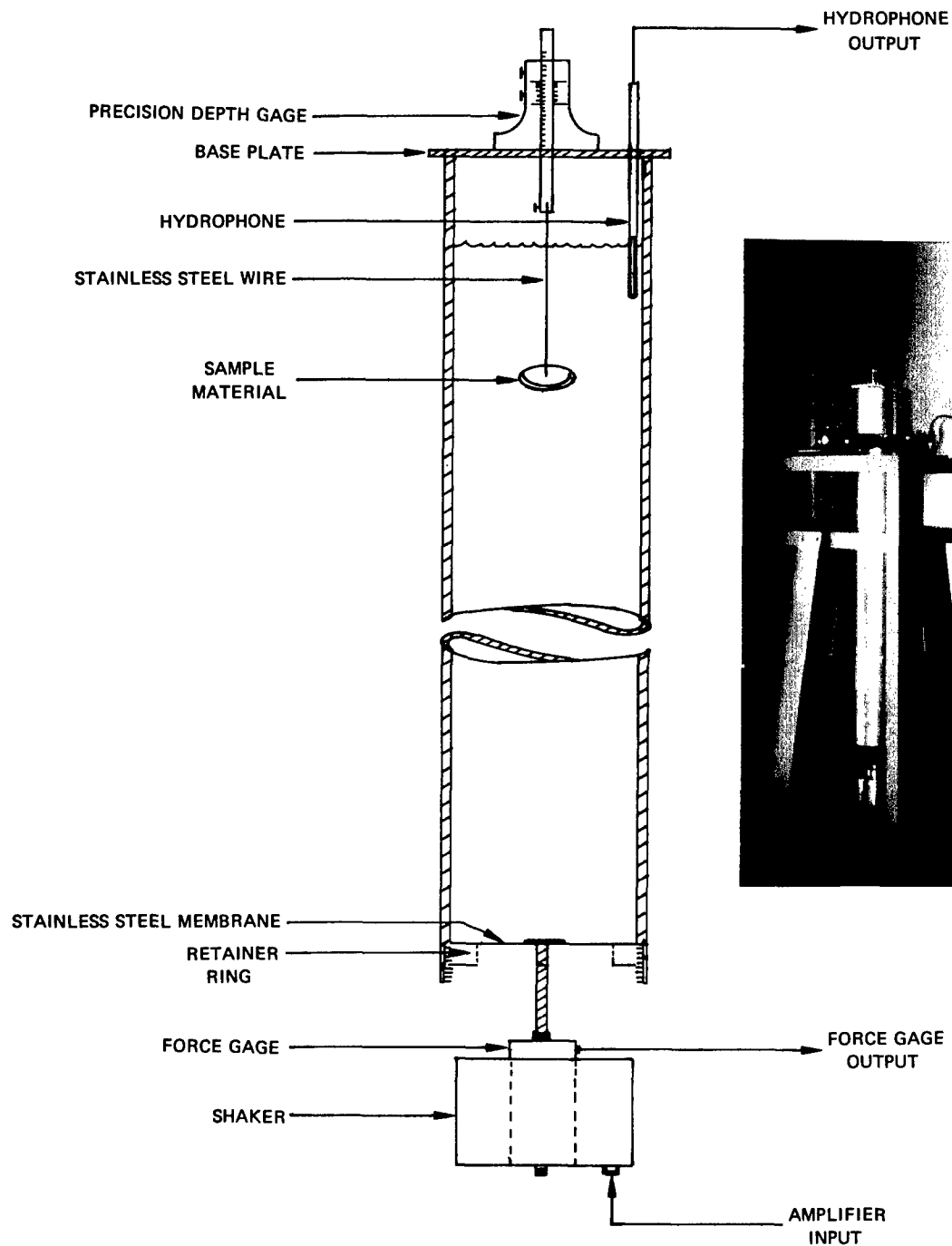


Figure 2 — Resonance Tube

The sample was lowered to the pressure antinode on a fine stainless steel wire, 0.057 in. in diameter, using a precision depth gage. The resonance frequency of the water column was measured with the wire inserted in the tube in order to take into account any effect the wire may have on the resonance properties of the tube. Before the sample was inserted into the water column, it was immersed in a wetting agent (such as a dilute solution of Aerosol) to prevent air bubbles from adhering to the sample. If air bubbles are present, a gradual increase in the stiffness of the sample will be observed until the bubbles disappear.

## EXPERIMENTAL RESULTS

The sound speed in the steel tube was found to be 92 percent of the free-field sound velocity, i.e.,  $1.37 \times 10^5$  cm/sec, at 25 C. This is in good agreement with the sound speed predicted by the improved Korteweg formula.<sup>7</sup>

In order to test the measurement system and to verify the theory, measurements were made of the bulk modulus of water, whose accepted value<sup>8</sup> is  $2.18 \times 10^{10}$  dyn/cm<sup>2</sup>. The "sample" was introduced simply by adding a known volume of water to the column. Five different volumes of water were added to the tube, and the shift in resonance frequency was noted each time. The bandwidth at the half-power point remained constant for each measurement, indicating no change in the losses of the tube. From the bulk modulus relation in Equation (27), it is seen that the shift in resonance frequency varies linearly with the change in water column height; hence, the data were fitted to a straight line by a linear least-squares method. The slope of this line (Figure 4) was then used to calculate a bulk modulus of  $2.18 \pm 0.04 \times 10^{10}$  dyn/cm<sup>2</sup> for water, agreeing within the experimental error with the accepted value. In Figure 5 straight lines were fitted to the data for various other materials.

This statistical smoothing minimized the experimental error and indicated confidence in the data, i.e., confidence in the data is determined by the "goodness" of fit of the data to a straight line. This technique was used for samples which were available in a variety of sizes. The results for various well known materials are compared in Table 1 with values found in the literature on dynamic testing.<sup>2,3,5</sup> All the values agree within the experimental error with the published data.

## DISCUSSION

The work reported here describes a simple dynamic means for determining the bulk modulus and elastic loss factor of polymers from acoustic measurements. Actual tests were performed only on a few well-known materials at room temperature to verify the theory. However, these tests have shown that a great deal of information about the acoustic and mechanical properties of these materials can be obtained from this technique.

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<sup>8</sup>Kinsler, L.E. and A.R. Frey, "Fundamentals of Acoustics," Second Edition, John Wiley and Sons, Inc., New York, p. 196 (1962).

These studies are being extended to include various new composite materials such as fiber-reinforced epoxy resins, which have great resilience to static and dynamic loading and environmental corrosion and also exhibit desirable acoustical properties. Dynamic studies should provide a great wealth of information concerning the properties of these materials and their suitability for deep-ocean applications.

### **ACKNOWLEDGMENTS**

The author wishes to express his appreciation to Mr. G.R. Castellucci for helping set up and modify the instrumentation, to Dr. W.T. Reader for his invaluable suggestions, and to Dr. W.S. Cramer for many fruitful discussions.

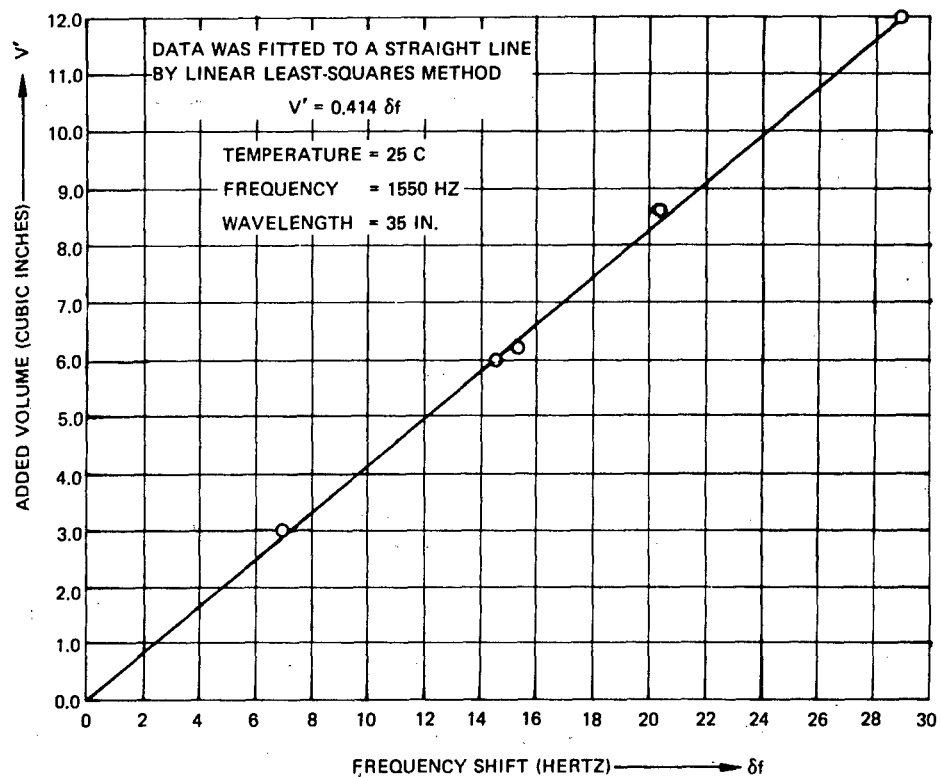


Figure 4 – Study of Added Volume versus Frequency Shift for Water

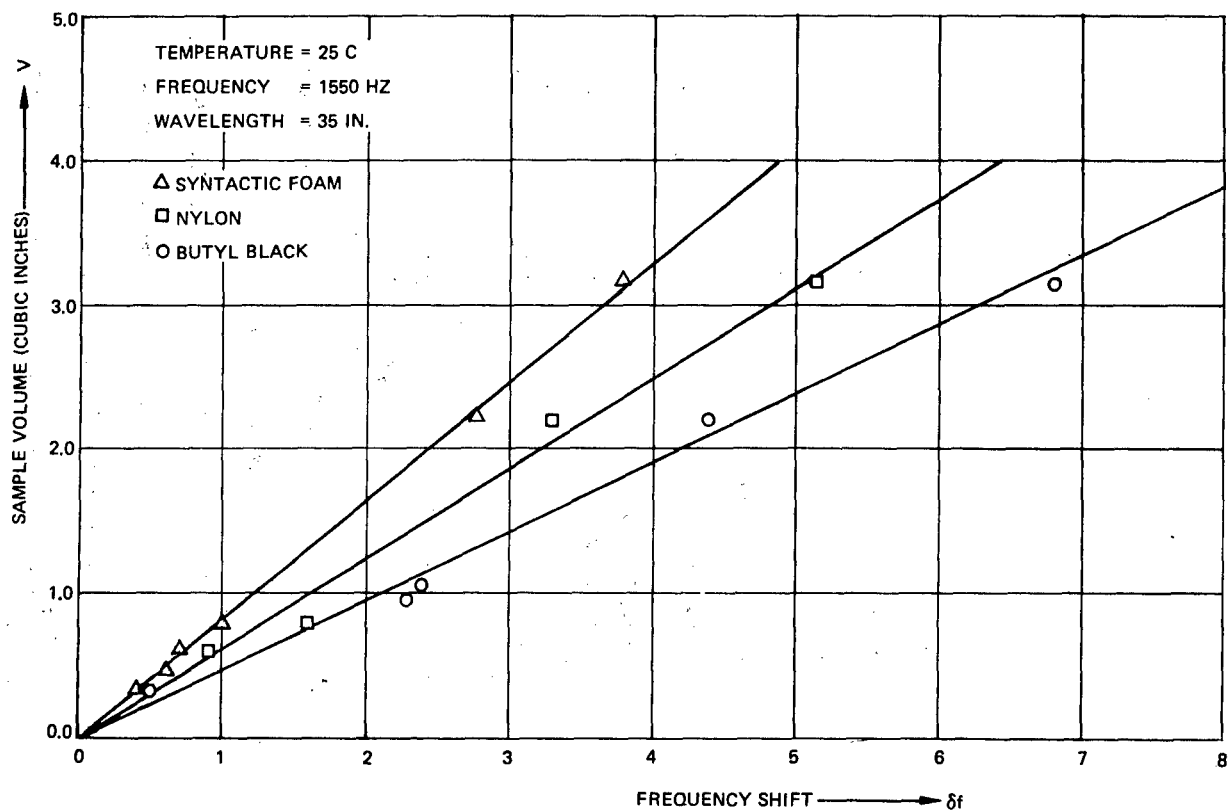


Figure 5 – Sample Volume versus Frequency Shift for Various Materials



TABLE 1 - BULK MODULUS AND ASSOCIATED ELASTIC LOSS FACTOR AT 1550 HERTZ  
AND 25 DEGREES CENTIGRADE

Material	Density $\rho$ , gm/cm <sup>3</sup>	Durometer Shore-Type A	Loss Factor $\eta$	From Resonant Water Column Method Bulk Modulus $B'$ , X $10^{-10}$ dyn/cm <sup>2</sup>	From Other Dynamic Methods Bulk Modulus $B'$ , X $10^{-10}$ dyn/cm <sup>2</sup>
Corprene*	0.60	45	0.285	0.026	-
Nlon 6**	1.14	-	0.007	3.75	-
Polycarbonate**	1.20	-	0.008	3.80	-
Polystyrene**	1.04	-	0.003	3.85	3.75
Polyurethane***	1.00	55	0.048	3.02	-
Polyvinylchloride**	1.38	-	0.009	3.98	3.58 - 4.48
Rho-c Rubber:					
Butyl Black†	1.16	40	0.050	2.47	2.56
Neoprene	1.22	60	0.010	2.62	2.65
Nitrile††	1.15	60	0.033	2.44	2.57
Syntactic Foam:					
Read Plastic**	0.50	-	0.003	4.32	-
Naval Research Laboratory†††	0.65	-	0.000	4.89	-

\* Potomac Rubber Company Inc., Washington, D.C.

\*\* Read Plastics Inc., Washington, D.C.

\*\*\* Emerson and Cuming Inc., Dielectric Materials Division, Canton, Massachusetts

† B.F. Goodrich Aerospace and Defense Products, Akron 18, Ohio

†† Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland

††† Naval Research Laboratory, Underwater Sound Reference Division, Orlando, Florida

## APPENDIX A DERIVATION OF EQUATION (9)

Equation (9) can be derived by considering the condensation of the elastic material as a longitudinal wave propagates through it and the thermodynamic process involved when the material is condensed.

The condensation of the elastic material is defined by<sup>8</sup>

$$s = \frac{\rho' - \rho_0'}{\rho_0'} \approx -\frac{\partial \xi}{\partial x}$$

where  $\rho'$  is the instantaneous density at any point

$\rho_0'$  is the constant equilibrium density of the material and

$\xi$  is the particle displacement from the equilibrium position along the axis of the rigid cylinder.

This equation is also known as the equation of continuity.

We may assume the thermodynamic process involved is adiabatic because little or no heat energy is transferred to the surrounding fluid. For an adiabatic process the acoustical pressure is given by

$$p = \rho_0' c'^2 s$$

where  $c'$  is the sound speed in the material.

Replacing  $s$  with its equivalent  $-\frac{\partial \xi}{\partial x}$  and  $\rho_0' c'^2$  by the complex bulk modulus  $B^*$  yields

$$p = -B^* \frac{\partial \xi}{\partial x} \quad (28)$$

The particle velocity may be expressed as

$$u = \frac{\partial \xi}{\partial t} = i \omega \xi$$

or

$$\frac{\partial \xi}{\partial x} = \frac{1}{i \omega} \frac{\partial u}{\partial x}$$

Substituting this expression into Equation (28) yields

$$p = -\frac{B^*}{i \omega} \frac{\partial u}{\partial x} \quad (29)$$

If we assume that the sample is a thin layer, as shown in Figure 6:

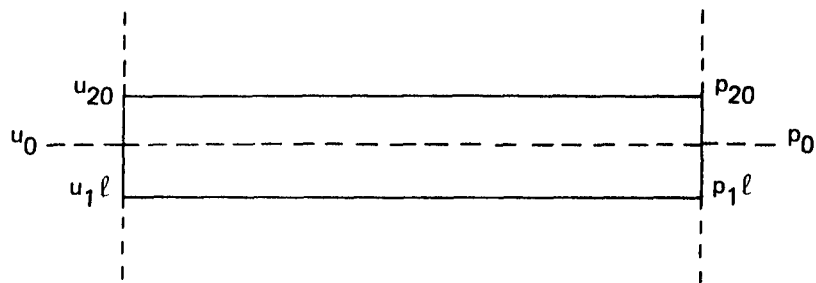


Figure 6 – Thin Layered Sample

Expressing  $u_{20}$  and  $u_{1\ell}$  in terms of  $u_0$ :

$$u_{20} \approx u_0 + \left( \frac{\partial u}{\partial x} \right)_0 \frac{D}{2}$$

and

$$u_{1\ell} \approx u_0 - \left( \frac{\partial u}{\partial x} \right)_0 \frac{D}{2}$$

Therefore

$$(u_{20} - u_{1\ell}) \approx \left( \frac{\partial u}{\partial x} \right)_0 D$$

or

$$\left( \frac{\partial u}{\partial x} \right)_0 \approx \left( \frac{u_{20} - u_{1\ell}}{D} \right)$$

From Equation (29), applied at  $x = 0$

$$p_0 = -\frac{B^*}{i\omega} \left( \frac{\partial u}{\partial x} \right)_0 \approx -\frac{B^*}{i\omega} \left( \frac{u_{20} - u_{1\ell}}{D} \right)$$

and when  $D \ll \lambda$  ( $\lambda$  = wavelength),  $p_0 \approx p_{1\ell} \approx p_{20}$ , therefore

$$p_{1\ell} = p_{20} = \frac{B^*}{i\omega D} (u_{1\ell} - u_{20}) \quad (9)$$

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13. ABSTRACT  A dynamic measurement technique is presented for determining the bulk modulus and the associated-loss factor of elastomeric and plastic materials. A resonant water column is used from which stiffness and damping are determined by the resonance frequency and the bandwidth of its resonance curve. A sample is inserted into the water column at a pressure maximum, and the resonance frequency and the bandwidth of the system are measured. The difference in the resonance frequency and the bandwidth of the water column without and with the sample present is then used to calculate the real component of the bulk modulus and the loss factor of the sample. Experimental results for various materials are presented which verify the theory.			

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Acoustic Measuring of Polymers (1) Dynamic Bulk Modulus of Polymers (2) Elastic-Loss Factor of Polymers Polymers, acoustical measuring Elastomeric and Plastic Materials, Acoustical Measuring Elastomeric and Plastic Materials, determining dynamic bulk modulus and elastic loss factors						